



The Role of Charge Order in the Mechanism of High Temperature Superconductivity

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References

Enrico Arrigoni, Eduardo Fradkin and Steven A. Kivelson, *Mechanism of High Temperature Superconductivity in a striped Hubbard Model*, Phys. Rev. B **69**, 214519 (2004);
arXiv:cond-mat/0309572

Steven A. Kivelson, Eduardo Fradkin, Vadim Oganesyan, Ian Bindloss, John Tranquada, Aharon Kapitulnik and Craig Howald, *How to detect fluctuating stripes in high temperature superconductors*, Rev. Mod. Phys. **75**, 1201 (2003); arXiv:cond-mat/02010683.

Steven A. Kivelson, Eduardo Fradkin and Victor J. Emery, *Electronic Liquid Crystal Phases of a Doped Mott Insulator*, Nature **393**, 550 (1998); arXiv:cond-mat/9707327.

Victor J. Emery, Eduardo Fradkin, Steven A. Kivelson and Tom C. Lubensky,, *Quantum Theory of the Smectic Metal State in Stripe Phases*, Phys. Rev. Lett. **85** 2160 (2000); arXiv:cond-mat/0001077

Mats Granath, Vadim Oganesyan , Steven A. Kivelson, Eduardo Fradkin and Victor J. Emery, *Nodal quasi-particles and coexisting orders in striped superconductors*, Phys. Rev. Lett. **87** 167011 (2001);
arXiv:cond-mat/0010350.

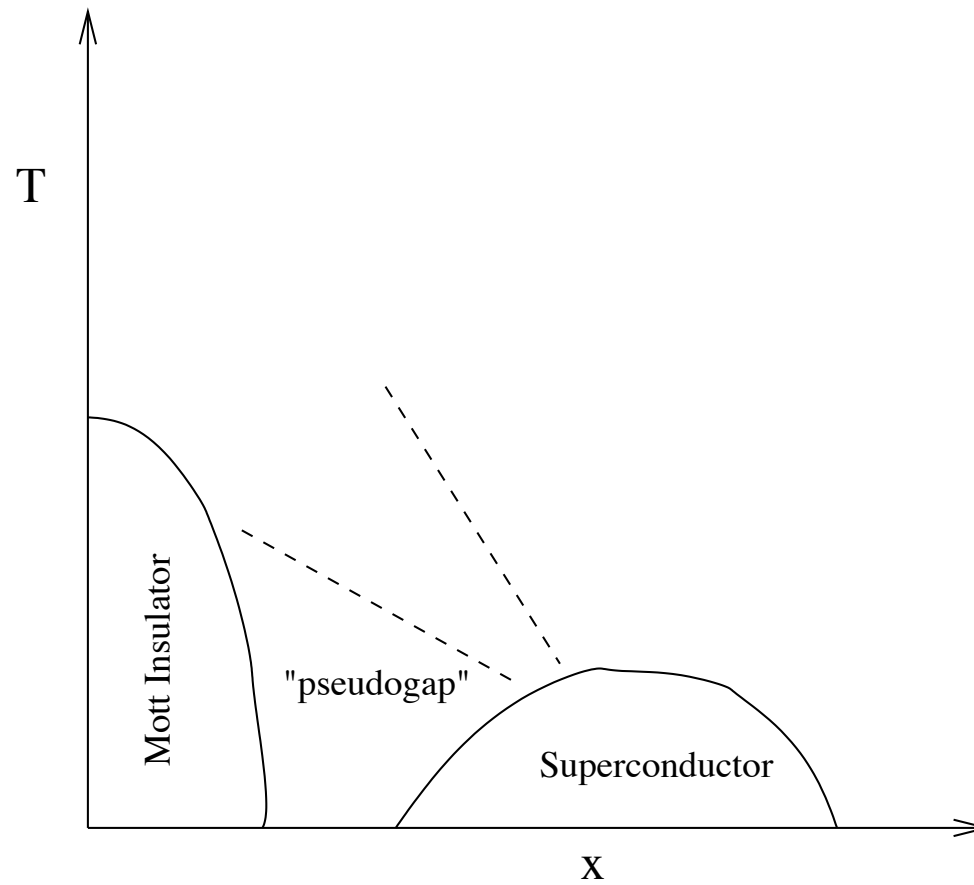
Outline

- Inhomogeneous Phases are **inherent** to Doped Mott Insulators
- Charge Order and Superconductivity: **friend or foe?**
- **High T_c Superconductivity** in a Striped Hubbard Model
- Isolated doped Hubbard Ladders as the **prototype spin-gap systems**
- Estimating T_c : **How high is high?**
- Conclusions and Open Questions

Punch Line

- Long held belief: charge order competes and suppresses superconductivity
- Electronic liquid crystal phases, not only can coexist with superconductivity but can also provide a mechanism for high T_c superconductivity.
- Inhomogeneous phases: natural local pairing mechanism with purely repulsive interactions
- This mechanism is not due to an infinitesimal instability
- Underlying normal state is not a Fermi liquid and it does not have quasiparticles
- T_c scales like a simple power of a coupling instead of an exponential dependence

Phase Diagram of the High T_c Superconductors



What do experiments tell us

- Evidence for **stripe charge order** in underdoped high temperature superconductors (LSCO, non-SC LBCO and YBCO) (Tranquada, Ando, Mook)
- Evidence of **coexistence of *fluctuating* stripe charge order and superconductivity** in LSCO and YBCO (Mook, Tranquada)
- Similarity of **high energy neutron spectra in LSCO, non-SC LBaCO, YBCO and the ladder system** (Tranquada, Mook, Keimer, Buyers)
- Evidence of **induced charge order** in the SC phase in vortex halos: neutrons in LSCO (B. Lake), STM in BSCCO (Davis)
- STM Experiments: **short range stripe order** (on scales **long compared to ξ_0**), possible broken rotational symmetry (BSCCO and NaCCOC) (Kapitulnik, Davis, Yazdani)
- Transport experiments give evidence for **charge domain switching in YBCO wires** (Van Harlingen and Weissmann) and **hysteretic effects** in the “normal state” of LSCO (Panagopoulos)

Electron Liquid Crystal Phases

S. Kivelson, E. Fradkin, V. Emery, Nature **393**, 550 (1998)

Doping a Mott insulator: inhomogeneous phases arise due to the competition between **phase separation** and **strong correlations**

- **Crystal Phases**: break all continuous translation symmetries and rotations
- **Smectic (Stripe) phases**: break one translation symmetry and rotations
- **Nematic and Hexatic Phases**: are uniform and anisotropic
- **Uniform fluids**: break no spatial symmetries

High T_c Superconductors : Lattice effects \Rightarrow breaking of point group symmetries

If lattice effects are weak (high T) \Rightarrow **continuous symmetries essentially recovered**

2DEG in GaAs heterostructures \Rightarrow continuous symmetries

Electronic Liquid Crystal Phases in High T_c Superconductors

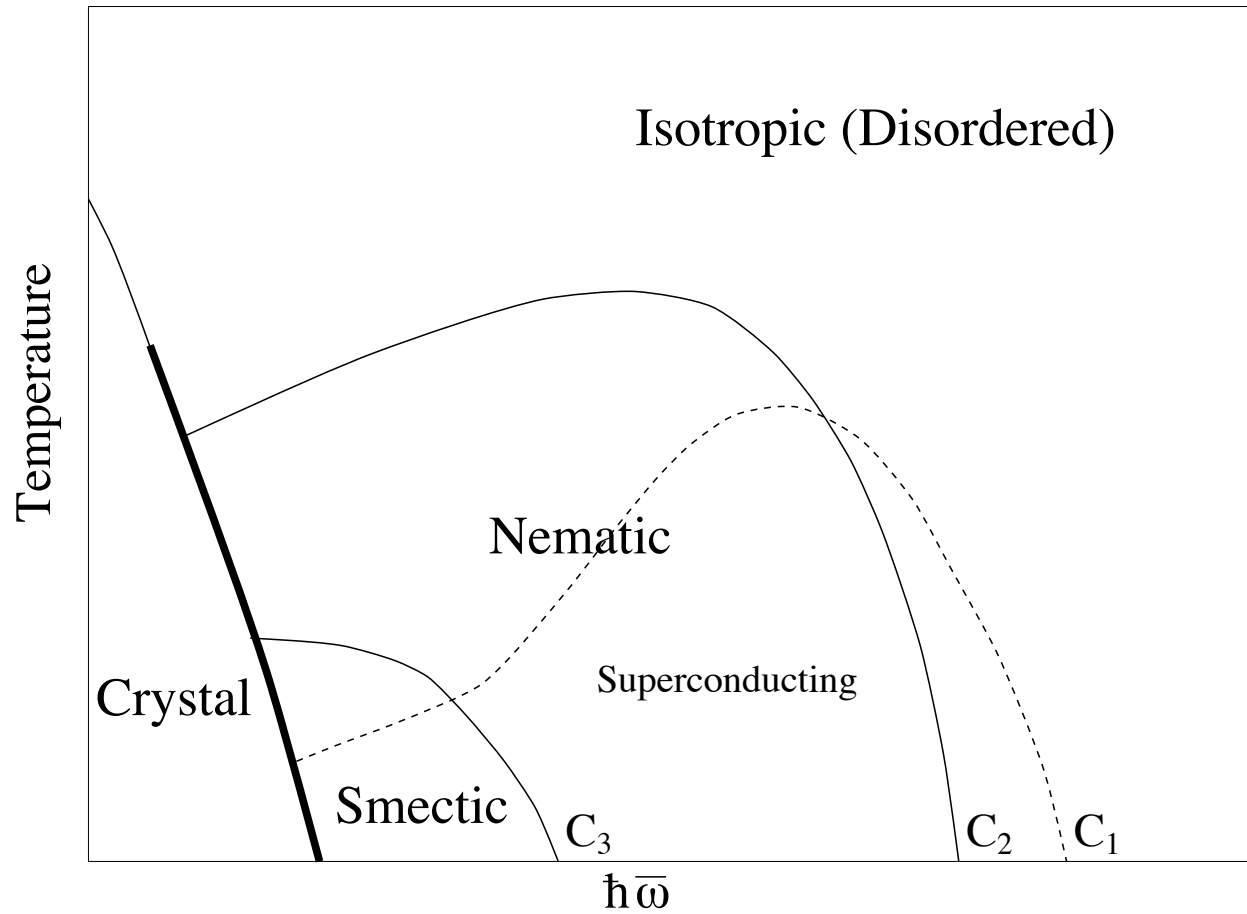
- **Liquid**: Isotropic, breaks no spacial symmetries; either a conductor or a superconductor
- **Nematic**: Lattice effects reduce the symmetry to a rotations by $\pi/2$ (“Ising”); translation and reflection symmetries are unbroken; it is an anisotropic liquid with a preferred axis
- **Smectic**: breaks translation symmetry only in one direction but liquid-like on the other; Stripe phase; (infinite) anisotropy of conductivity tensor
- **Crystal(s)**: electron solids (“CDW”); insulating states.

Soft Quantum Matter

or

Quantum Soft Matter

Schematic Phase Diagram of Doped Mott Insulators



Order Parameters for the Smectic (Stripe) State

- Two-dimensionally ordered state with unidirectional CDW order
- charge modulation \Rightarrow charge stripe
- if it coexists with spin order \Rightarrow spin stripe
- stripe state \Rightarrow new Bragg peaks of the electron density at

$$\vec{k} = \pm \vec{Q}_{\text{ch}} = \pm \frac{2\pi}{\lambda_{\text{ch}}} \hat{e}_x$$

- spin stripe \Rightarrow magnetic Bragg peaks at

$$\vec{k} = \vec{Q}_s = (\pi, \pi) \pm \frac{1}{2} \vec{Q}_{\text{ch}}$$

- **Order Parameter:** $\langle n_{\vec{Q}_{\text{ch}}} \rangle$, Fourier component of the electron density at \vec{Q}_{ch} .

Stripe Phases and the Mechanism of high T_c superconductivity in Strongly Correlated Systems

Since the discovery of high T_c superconductivity it has been clear that

- High T_c Superconductors are never normal metals and **don't have well defined quasiparticles in the "normal state"** (linear resistivity, ARPES)
- the "parent compounds" are strongly correlated Mott insulators
- repulsive interactions dominate
- whatever "the mechanism" is has to account for these facts

Problem

BCS is so successful in conventional metals that the term **mechanism** naturally evokes the idea of a **weak coupling instability** with (write here your favorite boson) mediating an attractive interaction between **well defined quasiparticles**.

Superconductivity in a Doped Mott Insulator

or

How To Get Pairing from Repulsive Interactions

- **Universal assumption:** 2D Hubbard-like models should contain the essential physics
- **“RVB” mechanism:**
 - Mott insulator: spins are bound in singlet valence bonds; it is a strongly correlated spin liquid, essentially a **pre-paired insulating state**
 - **spin-charge separation in the doped state leads to high T_c superconductivity**

Problems

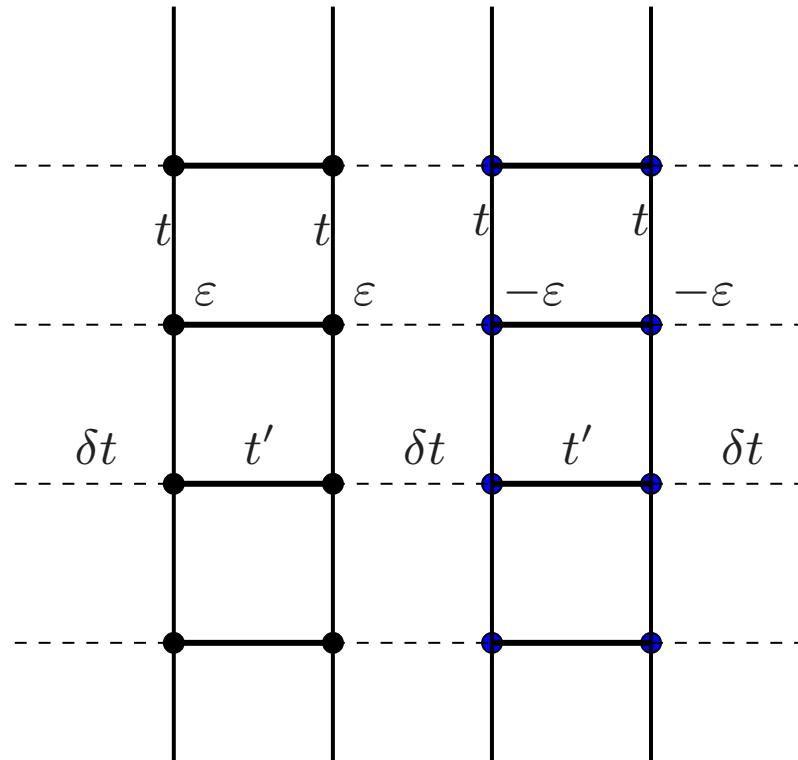
- there is no real evidence that the simple 2D Hubbard model favors superconductivity (let alone high T_c superconductivity)
- all evidence indicates that if anything it wants to be an insulator and to phase separate (finite size diagonalizations, various Monte Carlo simulations)
- strong tendency for the ground states to be inhomogeneous and possibly anisotropic
- no evidence (yet) for a spin liquid in 2D Hubbard-type models

Why an Inhomogeneous State is Good for high T_c superconductivity

Stripe Hubbard Model

A Cartoon of the Strongly Correlated Stripe Phase

$$H = - \sum_{\langle \vec{r}, \vec{r}' \rangle, \sigma} t_{\vec{r}, \vec{r}'} \left[c_{\vec{r}, \sigma}^\dagger c_{\vec{r}', \sigma} + \text{h.c.} \right] + \sum_{\vec{r}, \sigma} \left[\epsilon_{\vec{r}} c_{\vec{r}, \sigma}^\dagger c_{\vec{r}, \sigma} + \frac{U}{2} c_{\vec{r}, \sigma}^\dagger c_{\vec{r}, -\sigma}^\dagger c_{\vec{r}, -\sigma} c_{\vec{r}, \sigma} \right]$$



Physics of the 2-leg ladder

White, Affleck and Scalapino; Noack, White and Scalapino; Siller, Troyer and Rice; Balents and Fisher; Dagotto and Rice; Emery, Kivelson and Zachar; Wu, Liu and Fradkin

We will consider first an **isolated 2-leg ladder**: here we set $\delta t = 0$

- At $x = 0$ there is a **unique fully gapped ground state** (“C0S0”); for $U \gg t$, $\Delta_s \sim J/2$
- For $0 < x < x_c \sim 0.3$, **Luther-Emery liquid**: no charge gap and large spin gap (“C1S0”); **spin gap $\Delta_s \downarrow$ as $x \uparrow$** , and $\Delta_s \rightarrow 0$ as $x \rightarrow x_c$
- **Effective Hamiltonian for the charge degrees of freedom**

$$H = \int dy \frac{v_c}{2} \left[K (\partial_y \theta)^2 + \frac{1}{K} (\partial_x \phi)^2 \right] + \dots$$

ϕ : CDW phase field; θ : SC phase field; $[\phi(y'), \partial_y \theta(y)] = i\delta(y - y')$

- x -dependence of Δ_s , K , v_c , and μ depends on t'/t and U/t
- ... represent cosine potentials: Mott gap Δ_M at $x = 0$
- **Excitations for $x \rightarrow 0$ are spinless charge $2e$ fermionic solitons**

- $K \rightarrow 2$ as $x \rightarrow 0$; $K \sim 1$ for $x \sim 0.1$, and $K \sim 1/2$ for $x \sim x_c$
- $\chi_{\text{SC}} \sim \Delta_s/T^{2-K^{-1}}$ $\chi_{\text{CDW}} \sim \Delta_s/T^{2-K}$
- $\chi_{\text{CDW}}(T) \rightarrow \infty$ and $\chi_{\text{SC}}(T) \rightarrow \infty$ for $0 < x < x_c$
- For $x \lesssim 0.1$, $\chi_{\text{SC}} \gg \chi_{\text{CDW}}!$
- **Electron** ($P = 2\pi x$)

$$\Psi_{\pm, \sigma}^{\dagger} \sim \exp\left(i\sqrt{\pi/2}[\theta \pm \phi + \sigma\theta_s \pm \sigma\phi_s] \pm iP_y/2\right)$$

CDW

$$\hat{\rho}_P = \sum_{\sigma} \Psi_{L, \sigma}^{\dagger} \Psi_{R, \sigma} \propto \exp(iPy + i\sqrt{2\pi}\phi)$$

SC

$$\hat{\Phi} = \Psi_{L, \uparrow}^{\dagger} \Psi_{R, \downarrow}^{\dagger} + \Psi_{R, \uparrow}^{\dagger} \Psi_{L, \downarrow}^{\dagger} \propto \exp\left(1\sqrt{2\pi}\theta\right)$$

Effects of Inter-ladder Couplings

- In the Luther-Emery phase, $0 < x < x_c$, there is a spin gap and **single particle tunneling is irrelevant**
- Second order processes in δt :
 - **marginal** (and small) forward scattering inter-ladder interactions
 - Josephson couplings, **possibly relevant**
 - CDW couplings, **possibly relevant**

- **Relevant Perturbations**

$$H' = - \sum_J \int dy \left[\mathcal{J} \cos \left(\sqrt{2\pi} \Delta\theta_J \right) + \mathcal{V} \cos \left(\Delta P_J y + \sqrt{2\pi} \Delta\phi_J \right) \right]$$

J : ladder index; $P_J = 2\pi x_J$, $\Delta\phi_J = \phi_{J+1} - \phi_J$, etc.

- \mathcal{J} and \mathcal{V} are effective couplings which must be computed from microscopics
- Estimate: $\mathcal{J} \approx \mathcal{V} \propto (\delta t)^2 / J$

Period 2 works for $x \ll 1$

- If all the ladders are equivalent, a period 2 stripe ordered or column state
- For an isolated ladder $T_c = 0$
- $\mathcal{J} \neq 0$ and $\mathcal{V} \neq 0$, $T_C > 0$
- For $x \lesssim 0.1$ **CDW couplings are irrelevant** ($1 < K < 2$): Inter-ladder Josephson coupling leads to a superconducting state in a restricted range of small x with rather low T_c .

$$2\mathcal{J}\chi_{SC}(T_c) = 1$$

- $T_c \propto \delta t x$
- For larger x , $K < 1$ and χ_{CDW} is **more strongly divergent** than χ_{SC}
- CDW couplings become more relevant \Rightarrow Insulating, incommensurate CDW state with ordering wave number $P = 2\pi x$.

Why Period 4 works!

- Consider an alternating array of A and B type ladders (with different electron affinities) in the LE regime

- SC T_c :

$$(2\mathcal{J})^2 \chi_{\text{SC}}^A(T_c) \chi_{\text{SC}}^B(T_c) = 1$$

- CDW T_c :

$$(2\mathcal{V})^2 \chi_{\text{CDW}}^A(P, T_c) \chi_{\text{CDW}}^B(P, T_c) = 1$$

- 2D CDW order is greatly suppressed due to the mismatch between ordering vectors, P_A and P_B , on neighboring ladders
- For inequivalent ladders SC beats CDW if

$$2 > K_A^{-1} + K_B^{-1} - K_A; \quad 2 > K_A^{-1} + K_B^{-1} - K_B$$

-

$$T_c \sim \Delta_s \left(\frac{\mathcal{J}}{\widetilde{W}} \right)^\alpha; \quad \alpha = \frac{2K_A K_B}{[4K_A K_B - K_A - K_B]}$$

- $\mathcal{J} \sim \delta t^2 / J$ and $\widetilde{W} \sim J$; T_c is (power law) small for small \mathcal{J} ! ($\alpha \sim 1$).

How reliable are these estimates?

- This is a **mean-field estimate** for T_c and it is an **upper bound** to the actual T_c .
- T_c should be **suppressed by phase fluctuations** by up to a factor of 2.
- Indeed, **perturbative RG studies** for small \mathcal{J} yield the **same power law dependence**. This result is **asymptotically exact** for $\mathcal{J} \ll \widetilde{W}$.
- Since T_c is a smooth function of $\delta t/\mathcal{J}$, it is reasonable to **extrapolate for $\delta t \sim \mathcal{J}$** .
- $\Rightarrow T_c^{\max} \propto \Delta_s \Rightarrow$ **high T_c** .
- This is in contrast to the **exponentially small T_c** as obtained in a **BCS-like mechanism**.

Conclusions and Open Questions

- a period 2 modulation can produce superconductivity with a relatively low T_c in a restricted doping range,
- a period 4 modulation produces higher critical temperatures on a broader range of doping.
- T_c is only **power-law small**, with $\alpha \sim 1$
- no exponential suppression of $T_c \Rightarrow$ “high T_c ”
- This model is cartoon of the symmetry breaking of stripe (smectic) state
- It has a **large spin gap** and it does not have low-energy spin fluctuations
- **the order-parameter is d -wave like: it changes sign under $\pi/2$ rotations**
- It does not have **nodal fermionic excitations**
- **Nodal fermions may appear upon a Lifshitz transition at larger x**